



PAPER-1(B.E./B. TECH.)

JEE (Main) 2021

Questions & Solutions

(Reproduced from memory retention)

Date : 25 February, 2021 (SHIFT-1) Time ; (9.00 am to 12.00 pm)

Duration : 3 Hours | Max. Marks : 300

SUBJECT : MATHEMATICS

2nd Flood, Grand Plaza, Fraser Road, Dak Banglow, Patna - 800001

Tel. : + 8448446676 | Website : www.vidyapeethacademy.com

MATHEMATICS

1. If $x = \sum_{n=0}^{\infty} \cos^{2n}(\theta)$, $y = \sum_{n=0}^{\infty} \sin^{2n}(\phi)$, $\theta, \phi \in \left(0, \frac{\pi}{2}\right)$ and $Z = \sum_{n=0}^{\infty} (\sin \phi)^{2n} \cdot (\cos \theta)^{2n}$ then -

- (1) $Z = \frac{xy}{xy-1}$ (2) $Z = \frac{xy}{xy+1}$ (3) $Z = \frac{xy-1}{xy}$ (4) $Z = \frac{xy+1}{xy}$

Ans. (1)

Sol. $x = 1 + \cos^2\theta + \cos^4\theta + \dots = \frac{1}{1 - \cos^2\theta} = \frac{1}{\sin^2\theta}$

$y = 1 + \sin^2\phi + \sin^4\phi + \dots = \frac{1}{1 - \sin^2\phi} = \frac{1}{\cos^2\phi}$

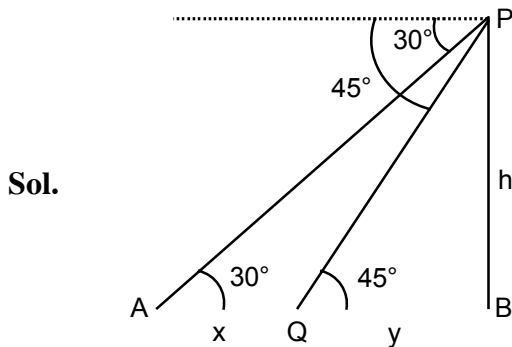
$z = 1 + \sin^2\phi \cos^2\theta + \sin^4\phi \cos^4\theta + \dots = \frac{1}{1 - \sin^2\phi \cos^2\theta} = \frac{1}{1 - \frac{1}{x} \frac{1}{y}}$

$\Rightarrow z = \frac{xy}{xy-1}$

2. A person is standing at P on a tower of height h. The angles of depression to the ground at A and Q are 30° & 45° respectively. Time taken to cover the distance AQ is 20s. Find the time to cover distance QB.

- (1) $10(\sqrt{3}-1)$ sec (2) 30 sec (3) $10(\sqrt{3}+1)$ sec (4) $10(\sqrt{2}+1)$ sec

Ans. (3)



In $\triangle ABP$

$\frac{h}{x+y} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

$x + y = \sqrt{3} h \dots(i)$

In $\triangle QBP$

$\frac{h}{y} = \tan 45^\circ = 1 \dots(ii)$

$h =$

$$x + y = \sqrt{3} y$$

$$x = (\sqrt{3} - 1)y$$

Let speed is v

$$\frac{x}{v} = 20 \Rightarrow x = 20v$$

$$\therefore 20v = (\sqrt{3} - 1)y$$

$$\text{Time to cover } y \text{ distance} = \frac{y}{v} = \frac{20}{\sqrt{3} - 1} = 10(\sqrt{3} + 1) \text{ sec}$$

3. Image of P(3,5) on the line $y = x + 1$ is Q. Then Q lies on

$$(1) (x - 4)^2 + (y - 2)^2 = 4$$

$$(2) (x - 1)^2 + y^2 = 4$$

$$(3) x^2 + y^2 = 4$$

$$(4) x^2 + (y - 2)^2 = 4$$

Ans. (1)

Sol. Image of P(3,5) on the line $x - y + 1 = 0$ is

$$\frac{x - 3}{1} = \frac{y - 5}{-1} = \frac{-2(3 - 5 + 1)}{2} = 1$$

$$x = 4, y = 4$$

\therefore Image is (4,4)

Which lies on

$$(x - 4)^2 + (y - 2)^2 = 4$$

4. If $A \rightarrow (B \rightarrow A)$ then which of the following is contrapositive

$$(1) A \rightarrow (B \vee A)$$

$$(2) (A \wedge B)$$

$$(3) (B \wedge \sim A) \rightarrow A$$

$$(4) (B \rightarrow A)$$

Ans. (1)

Sol. Contrapositive of $A \rightarrow (B \rightarrow A)$ is

$$\sim (B \rightarrow A) \rightarrow \sim A$$

$$(B \wedge \sim A) \rightarrow \sim A$$

5. Consider the parabola $y^2 = 6x$. If a tangent to the parabola is perpendicular to the line $2x + y = 1$, then which of the following point does not lie on the tangent:

$$(1) (5, 4)$$

$$(2) (4, 5)$$

$$(3) (6, 6)$$

$$(4) (8, 7)$$

Ans. (1)

Sol. Equation of tangent : $y = mx + \frac{3}{2m}$

$$m_T = \frac{1}{2} \quad (\because \text{perpendicular to line } 2x + y = 1)$$

$$\therefore \text{tangent is : } y = \frac{x}{2} + 3 \Rightarrow x - 2y + 6 = 0$$

6. Find the value of : $I = \int_{-1}^1 x^2 \cdot e^{[x^3]} dx$, where $[.]$ greatest integer function

- (1) $\frac{1}{3} - \frac{1}{3e}$ (2) $\frac{1}{3} + \frac{1}{3e}$ (3) $\frac{1}{3e} - \frac{1}{2}$ (4) 2

Ans. (2)

Sol. $I = \int_{-1}^0 x^2 \cdot e^{-1} dx + \int_0^1 x^2 dx$

$$\therefore I = \frac{x^3}{3e} \Big|_{-1}^0 + \frac{x^3}{3} \Big|_0^1$$

$$\Rightarrow I = \frac{1}{3e} + \frac{1}{3}$$

7. Let $A = \begin{bmatrix} 0 & \tan \frac{\theta}{2} \\ -\tan \frac{\theta}{2} & 0 \end{bmatrix}$ and $(I + A)(I - A)' = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then find the value of $13(a^2 + b^2)$

- (1) $26 \sec^2 \frac{\theta}{2}$ (2) $13 \tan^4 \frac{\theta}{2}$ (3) $26 \tan^2 \frac{\theta}{2}$ (4) $13 \sec^4 \frac{\theta}{2}$

Ans. (4)

Sol. $A = \begin{bmatrix} 0 & \tan \frac{\theta}{2} \\ -\tan \frac{\theta}{2} & 0 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$I + A = \begin{bmatrix} 1 & \tan \frac{\theta}{2} \\ -\tan \frac{\theta}{2} & 1 \end{bmatrix}, \quad I - A = \begin{bmatrix} 1 & -\tan \frac{\theta}{2} \\ \tan \frac{\theta}{2} & 1 \end{bmatrix}, \quad (I - A)' = \begin{bmatrix} 1 & \tan \frac{\theta}{2} \\ -\tan \frac{\theta}{2} & 1 \end{bmatrix}$$

$$\therefore (I + A)(I - A)' = \begin{bmatrix} 1 & \tan \frac{\theta}{2} \\ -\tan \frac{\theta}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan \frac{\theta}{2} \\ -\tan \frac{\theta}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 - \tan^2 \frac{\theta}{2} & 2 \tan \frac{\theta}{2} \\ -2 \tan \frac{\theta}{2} & 1 - \tan^2 \frac{\theta}{2} \end{bmatrix}$$

$$\therefore a = 1 - \tan^2 \frac{\theta}{2}, \quad b = -2 \tan \frac{\theta}{2}$$

$$\therefore 13(a^2 + b^2) = 13 \left(\left(1 - \tan^2 \frac{\theta}{2}\right)^2 + 4 \tan^2 \frac{\theta}{2} \right) = 13 \left(1 + \tan^2 \frac{\theta}{2}\right)^2 = 13 \sec^4 \frac{\theta}{2}$$

8. If $L_1 : (2 - i)z = 4i + (2 + i)\bar{z}$
 $L_2 : (2 + i)z = (i - 2)\bar{z}$; if L_1 & L_2 intersect at a point A and a circle is drawn with centre A touching the line $(i - 1)z + (1 + i)\bar{z} + 2i = 0$, then radius of circle is -

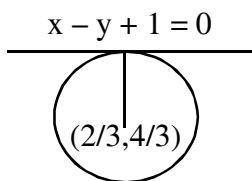
- (1) $\frac{3}{2\sqrt{2}}$ (2) $\frac{1}{\sqrt{2}}$ (3) $3\sqrt{2}$ (4) $\frac{1}{3\sqrt{2}}$

Ans. (4)

Sol. $L_1 = (2 - i)(x + iy) = 4i + (2 + i)(x - iy)$
 $\Rightarrow (2x + y) + i(2y - x) = (4 + x - 2y)i + (2x + y)$
 $\Rightarrow 2y - x = 4 + x - 2y$
 $\Rightarrow 4y - 2x = 4$
 $\Rightarrow x - 2y + 2 = 0$

$L_2 = (2 + i)(x + iy) = (i - 2)(x - iy)$
 $\Rightarrow 2x - y + i(2y + x) = (-2x + y) + i(x + 2y)$
 $\Rightarrow 2x - y = -2x + y$
 $\Rightarrow 2x - y = 0 \Rightarrow x - 2(2x) + 2 = 0$
 $\Rightarrow -3x + 2 = 0 \Rightarrow x = \frac{2}{3}, y = \frac{4}{3}$

$(i - 1)(x + iy) + (i + 1)(x - iy) + 2i = 0$
 $-x - y + x + y = 0, x - y + x - y + 2 = 0$



$$r = \left| \frac{\frac{2}{3} - \frac{4}{3} + 1}{\sqrt{2}} \right| = \left| \frac{1}{3\sqrt{2}} \right|$$

9. $\int \frac{\sin \theta \sin 2\theta (\sin^6 \theta + \sin^4 \theta + \sin^2 \theta) \sqrt{2 \sin^4 \theta + 3 \sin^2 \theta + 6}}{1 - \cos 2\theta} d\theta$ is equal to

- (1) $\frac{1}{6} [2\cos^6 \theta + 9\cos^4 \theta - 18 \cos^2 \theta + 11]^{3/2} + C$
 (2) $\frac{1}{18} [-2\cos^6 \theta + 9\cos^4 \theta - 18 \cos^2 \theta + 11]^{3/2} + C$
 (3) $\frac{1}{18} [2\cos^6 \theta - 9\cos^5 \theta + 18 \cos^2 \theta + 11]^{3/2} + C$
 (4) $\frac{1}{18} [-2\cos^6 \theta - 9\cos^5 \theta - 18 \cos^3 \theta + 11]^{3/2} + C$

Ans. (2)

Sol.
$$\int \frac{2 \sin^2 \theta \cos 2\theta (\sin^6 \theta + \sin^4 \theta + \sin^2 \theta) \sqrt{2 \sin^4 \theta + 3 \sin^2 \theta + 6}}{2 \sin^2 \theta} d\theta$$

Let $\sin \theta = t$ $\cos \theta d\theta = dt$

$$= \int (t^6 + t^4 + t^2) \sqrt{2t^4 + 3t^2 + 6} dt = \int (t^5 + t^3 + t) \sqrt{2t^6 + 3t^4 + 6t^2} dt$$

Let $2t^6 + 3t^4 + 6t^2 = z$

$12(t^5 + t^3 + t) dt = dz$

$$= \frac{1}{12} \int \sqrt{z} dz = \frac{1}{18} z^{3/2} + c$$

$$= \frac{1}{18} (2 \sin^6 \theta + 3 \sin^4 \theta + 6 \sin^2 \theta)^{3/2} + C$$

$$= \frac{1}{18} [(1 - \cos^2 \theta)(2(1 - \cos^2 \theta) + 3 - 3 \cos^2 \theta + 6)]^{3/2} + C$$

$$= \frac{1}{18} [(1 - \cos^2 \theta)(2 \cos^4 \theta - 7 \cos^2 \theta + 11)]^{3/2} + C$$

$$= \frac{1}{18} [-2 \cos^6 \theta + 9 \cos^4 \theta - 18 \cos^2 \theta + 11]^{3/2} + C$$

10. If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} + \hat{j}$, $\vec{c} = \hat{i} - 2\hat{j} - \hat{k}$ & $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$, $\vec{r} \cdot \vec{b} = 0$, then $\vec{r} \cdot \vec{a}$ is equal to

(1) $\frac{1}{2}$

(2) $\frac{3}{2}$

(3) $-\frac{3}{2}$

(4) $-\frac{1}{2}$

Ans. (2)

Sol. $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$

$(\vec{r} - \vec{c}) \times \vec{a} = \vec{0}$

$\Rightarrow \vec{r} - \vec{c} = \lambda \vec{a}$

$\vec{r} = \lambda \vec{a} + \vec{c}$

$\vec{r} \cdot \vec{b} = 0 \Rightarrow (\vec{a} \cdot \vec{b}) + (\vec{b} \cdot \vec{c}) = 0$

$2\lambda - 1 = 0 \Rightarrow \lambda = \frac{1}{2}$

$\therefore \vec{r} = \frac{1}{2} \vec{a} + \vec{c}$

$$\vec{r} = \frac{1}{2} (\hat{i} + \hat{j} - \hat{k}) + (\hat{i} - 2\hat{j} - \hat{k})$$

$$\vec{r} = \frac{3}{2} \hat{i} - \frac{3}{2} \hat{j} - \frac{3}{2} \hat{k}$$

$$\vec{r} = \frac{3}{2} (\hat{i} - \hat{j} - \hat{k})$$

$$\therefore \vec{r} \cdot \vec{a} = \frac{3}{2} (\hat{i} - \hat{j} - \hat{k}) \cdot (\hat{i} + \hat{j} - \hat{k}) = \frac{3}{2} (1 - 1 + 1) = \frac{3}{2}$$

11. Let f and g are defined from $\mathbb{N} \rightarrow \mathbb{N}$ such that $f(n + 1) = f(n) + f(1)$, then which of the following is not true?

- (1) f is one-one (2) f is onto $\Rightarrow f(x) = x$
 (3) if $f \circ g$ is one-one then g is onto (4) if $f \circ g$ is onto, then g is onto

Ans. (3)

Sol. $f(n + 1) = f(n) + 1$

$$f(2) = 2f(1)$$

$$f(3) = 3f(1)$$

$$f(4) = 4f(1)$$

....

$$f(n) = nf(1)$$

$f(x)$ is one-one

12. If the curves $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{c} + \frac{y^2}{d} = 1$ cut each other orthogonally, then :

- (1) $a - b = c - d$ (2) $ab = \frac{c+d}{a+b}$ (3) $a + b = c + d$ (4) $ab = \frac{c-d}{a-b}$

Ans. (1)

Sol. $\frac{x^2}{a} + \frac{y^2}{b} = 1$ (1)

$$\text{diff : } \frac{2x}{a} + \frac{2y}{b} \frac{dy}{dx} = 0 \Rightarrow \frac{y}{b} + \frac{dy}{dx} = \frac{-x}{a}$$

$$\frac{dy}{dx} = \frac{-bx}{ay} \quad \text{.....(2)}$$

$$\frac{x^2}{c} + \frac{y^2}{d} = 1 \quad \text{.....(3)}$$

$$\text{diff : } \frac{dy}{dx} = \frac{-d}{cy} \quad \text{.....(4)}$$

$$m_1 m_2 = -1 \Rightarrow \frac{-bx}{ay} \times \frac{-dx}{cy} = -1$$

$$\Rightarrow bdx^2 = -acy^2 \quad \text{.....(5)}$$

$$(1) - (3) \Rightarrow \left(\frac{1}{a} - \frac{1}{c}\right)x^2 + \left(\frac{1}{b} - \frac{1}{d}\right)y^2 = 0$$

$$\Rightarrow \frac{c-a}{ac}x^2 + \frac{d-b}{bd} \times \left(\frac{-bd}{ac}\right)x^2 = 0 \quad (\text{Using 5})$$

$$\Rightarrow (c-a) - (d-b) = 0$$

$$\Rightarrow c-a = d-b$$

$$\Rightarrow c-d = a-b$$

13. If : $\sin 2\theta + \tan 2\theta > 0$; $\theta \in [0, 2\pi]$. Then the complete set of values of 'θ' which satisfies

(1) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

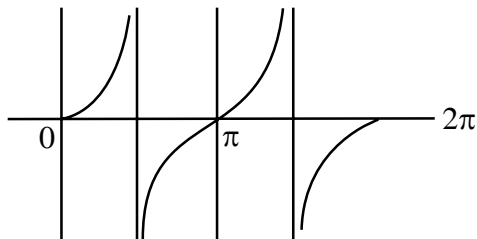
(2) $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{\pi}{2}, \frac{3\pi}{4}\right] \cup \left[\pi, \frac{5\pi}{4}\right] \cup \left[\frac{3\pi}{2}, \frac{7\pi}{4}\right]$

(3) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$

(4) None of these

Ans. (1)

Sol.



$$\tan 2\theta (1 + \cos 2\theta) > 0$$

$$2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$$

$$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$$

14. Let the quadratic equation $ax^2 + bx + c = 0$ where a,b,c are obtained by rolling the dice thrice. What is the probability that equation has equal roots.

(1) $\frac{5}{216}$

(2) $\frac{1}{72}$

(3) $\frac{1}{36}$

(4) $\frac{3}{216}$

Ans. (1)

Sol. $a, b, c \in \{1,2,3,4,5,6\}$

$$n(s) = 6 \times 6 \times 6 = 216$$

$$D = 0 \Rightarrow b^2 = 4ac$$

$$ac = \frac{b^2}{4} \quad \text{If } b = 2, ac = 1 \Rightarrow a = 1, c = 1$$

$$\text{If } b = 4, ac = 4 \Rightarrow a = 1, c = 4$$

$$a = 4, c = 1$$

$$a = 2, c = 2$$

$$\text{If } b = 6, ac = 9 \Rightarrow a = 3, c = 3$$

$$\therefore \text{probability} = \frac{5}{216}$$

15. Let $f(x)$ is a polynomial of 6th degree with leading co-efficient unity and $\lim_{x \rightarrow 0} \frac{f(x)}{x^3} = 1$. Also, $x = 1$ & $x = -1$ are points of extremas of $f(x)$, then find the value of $5f(2)$.
- (1) 144 (2) 146 (3) 148 (4) 150

Ans. (1)

Sol. $f(x) = x^6 + ax^5 + bx^4 + x^3$
 $\therefore f'(x) = 6x^5 + 5ax^4 + 4bx^3 + 3x^2$
 Roots 1 & -1
 $\therefore 6 + 5a + 4b + 3 = 0$ & $-6 + 5a - 4b + 3 = 0$ solving
 $a = -\frac{3}{5}$ $b = -\frac{3}{2}$
 $\therefore f(x) = x^6 - \frac{3}{5}x^5 - \frac{3}{2}x^4 + x^3$
 $\therefore 5 \cdot f(2) = 5 \left[64 - \frac{96}{5} - 24 + 8 \right] = 144$

16. If slope at any point to a curve is $\frac{dy}{dx} = \frac{x^2 - 4x + y + 8}{(x - 2)}$ and the curve passes through the origin then which of the following points also lies on the curve ?
- (1) (2, 4) (2) (2, -4) (3) (-2, -4) (4) (3, 1)

Ans. (2)

Sol. $\frac{dy}{dx} = \frac{(x-2)^2 + y + 4}{(x-2)} = (x-2) + \frac{y+4}{(x-2)}$
 Let $x - 2 = t \Rightarrow dx = dt$
 and $y + 4 = u \Rightarrow dy = du$
 $\frac{dy}{dx} = \frac{du}{dt}$
 $\frac{du}{dt} = t + \frac{u}{t} \Rightarrow \frac{du}{dt} - \frac{u}{t} = t$
 I.F = $e^{\int \frac{-1}{t} dt} = e^{-\ln t} = \frac{1}{t}$
 $u \cdot \frac{1}{t} = \int t \cdot \frac{1}{t} dt \Rightarrow \frac{u}{t} = t + c$
 $\frac{y+4}{x-2} = (x-2) + c$
 Passing through (0, 0)
 $c = 0$
 $\Rightarrow (y + 4) = (x - 2)^2$

17. Find the number of point where $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$ is non differentiable.

- (1) 2 (2) 3 (3) 4 (4) 0

Ans. (2)

Sol. Non differentiable at $x = -\frac{1}{2}, -2, 1$

18. $\lim_{n \rightarrow \infty} \left(1 + \frac{1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}}{n^2} \right)^n$ is equal to

- (1) $\frac{1}{e}$ (2) 1 (3) 0 (4) $\frac{1}{e^2}$

Ans. (2)

Sol. Let limit be L

$$\text{So } L = e^{\lim_{n \rightarrow \infty} \left(\frac{1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}}{n} \right)} = e^k \text{ (say)}$$

Now assume $n = 2^p + A, \lambda \in \{0, 1, 2, \dots, 2^p - 1\}$

$$\begin{aligned} \text{Now assume } & 1 + \left(\frac{1}{2} + \frac{1}{3} \right) + \left(\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} \right) + \dots + \left(\frac{1}{2^{p-1}} + \frac{1}{2^{p-1}+1} + \dots + \frac{1}{2^p-1} \right) \\ & + \left(\frac{1}{2^p} + \frac{1}{2^p+1} + \dots + \frac{1}{2^p+\lambda} \right) = S \end{aligned}$$

$$\text{So } S < 1 + \underbrace{\left(\frac{1}{2} + \frac{1}{2} \right)} + \underbrace{\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \right)} + \dots + \underbrace{\left(\frac{1}{2^p} + \frac{1}{2^p} + \dots + \frac{1}{2^p} \right)}_{(\lambda+1)\text{times}}$$

$$\Rightarrow S < \underbrace{1+1+1+\dots+1}_{p \text{ times}} + \frac{\lambda+1}{2^p} < p+1$$

$$\text{Hence } k \leq \lim_{n \rightarrow \infty} \frac{p+1}{2^p} = 0$$

$$\text{Also } S > \underbrace{\left(\frac{1}{n} + \frac{1}{n} + \frac{1}{n} + \dots + \frac{1}{n} \right)}_{n \text{ times}} = 1$$

$$\text{Hence } k \geq \lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

So $L = 1$

19. If the direction cosines of two lines satisfy the relations : $l + m = n$ & $l^2 + m^2 = n^2$ and ' α ' is the angle between them, then the value of $\sin^4 \alpha + \cos^4 \alpha$ is :

- (1) $\frac{3}{4}$ (2) $\frac{5}{8}$ (3) $\frac{1}{2}$ (4) $\frac{3}{8}$

Ans. (2)

Sol. $l^2 + m^2 + n^2 = 1$

$$\therefore 2n^2 = 1 \Rightarrow n = \pm \frac{1}{\sqrt{2}}$$

$$\therefore l^2 + m^2 = \frac{1}{2} \text{ \& } l + m = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{1}{2} - 2lm = \frac{1}{2}$$

$$\Rightarrow lm = 0 \quad \text{or} \quad m = 0$$

$$\therefore l = 0, m = \frac{1}{\sqrt{2}} \quad \text{or} \quad l = \frac{1}{\sqrt{2}}$$

$$\langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle \quad \text{or} \quad \langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle$$

$$\therefore \cos \alpha = 0 + 0 + \frac{1}{2} = \frac{1}{2}$$

$$\therefore \sin^4 \alpha + \cos^4 \alpha = 1 - \frac{1}{2} \sin^2 (2\alpha) = 1 - \frac{1}{2} \cdot \frac{3}{4} = \frac{5}{8}$$

20. Let $f(x) = x^3 - ax^2 + bx + 1$ defined in $[1, 2]$. Rolle's theorem is applied on $f(x)$ in $[1, 2]$ such that

$f'\left(\frac{4}{3}\right) = 0$ then ordered pair (a, b) is

- (1) (-5, 8) (2) (5, 8) (3) (5, -8) (4) (-5, -8)

Ans. (2)

Sol. $f(1) = f(2)$

$$\Rightarrow 1 - a + b + 1 = 8 - 4a + 2b + 1$$

$$3a - b = 7 \quad \dots\dots\dots(1)$$

$$f'(x) = 3x^2 - 2ax + b$$

$$\Rightarrow f'\left(\frac{4}{3}\right) = 0 \Rightarrow 3 \times \frac{16}{9} - \frac{8}{3}a + b = 0$$

$$\Rightarrow -8a + 3b = -16 \quad \dots\dots\dots(2)$$

$$a = 5, b = 8$$

21. If $kx + 2y + 3z = 4$, $x - y - z = 5$, $10x - y - 2z = 9$ has infinite solutions then find the value of $|k|$

Ans. (11)

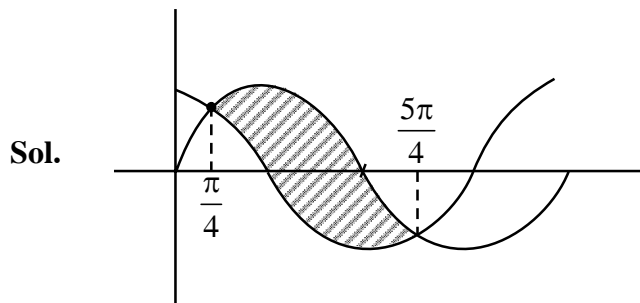
Sol. $D = \begin{vmatrix} k & 2 & 3 \\ 1 & -1 & -1 \\ 10 & -1 & -2 \end{vmatrix} = k(2(-1) - 2(-2 + 10)) + 3(-1 + 10)$

$$k - 16 + 27 = 0$$

$$k = -11$$

22. $y = \sin x$ and $y = \cos x$ intersect at many points. If area enclosed by them between two consecutive intersection points is A find A^4 .

Ans. 64



$$A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx = [-\cos x - \sin x]_{\pi/4}^{5\pi/4}$$

$$= -\left[\left(\cos \frac{5\pi}{4} + \sin \frac{\pi}{4}\right) - \left(\cos \frac{\pi}{4} + \sin \frac{\pi}{4}\right)\right]$$

$$= -\left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$\Rightarrow A^4 = (2\sqrt{2})^4 = 64$$

23. If $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$ and $A^2 = I$ & $xyz = 2$, $x + y + z > 0$

Find the value of $x^3 + y^3 + z^3$

Ans. 7

Sol. $A^2 = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix} \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow x^2 + y^2 + z^2 = 1$

$\Rightarrow x + y + z = 1$

$\Rightarrow xy + yz + zx = 0$

$|A|^2 = |I| \Rightarrow |A| = \pm 1 \Rightarrow 3xyz - (x^3 + y^3 + z^3) = \pm 1$

$x^3 + y^3 + z^3 = 3.2 \pm 1 = 7, 5$

$\Rightarrow x^3 + y^3 + z^3 = 7$

24. Let $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0 \forall x \in \mathbb{R}$ then integral value of k is

Ans. 3

Sol. $D < 0$

$(2(3k - 1))^2 - 4(8k^2 - 7) < 0$

$4(9k^2 - 6k + 1) - 4(8k^2 - 7) < 0$

$k^2 - 6k + 8 < 0$

$(k - 4)(k - 2) < 0$

$2 < k < 4$

then $k = 3$

25. The chance that a missile is intercepted is $\frac{1}{3}$. If missile is not intercepted the chance that it hits the target is $\frac{3}{4}$. the probability that all three missiles hit the target is (assume that launch of missiles are independent)

Ans. 0.125

Sol. Prob. = $\left(\frac{2}{3} \cdot \frac{3}{4}\right)^3 = \frac{1}{8}$

26. How many numbers from 100 to 1000 using the digits 1, 2, 3, 4, 5 which are divisible by 3 or 5 (No repetition)

(1) 36

(2) 32

(3) 30

(4) 28

Ans. (2)

Sol.

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 divisible by $\rightarrow 3$

$12 \rightarrow 3, 4, 5 \rightarrow 3! = 6$

$15 \rightarrow 2, 3, 4 \rightarrow 3! = 6$

$24 \rightarrow 1, 3, 5 \rightarrow 3! = 6$

$42 \rightarrow 1, 2, 3 \rightarrow 3! = 6$

24

divisible by 5

		5
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 = 12

4×3

Required No. = $24 + 12 - 4 = 32$